Wireless Network-Level Partial Relay Cooperation

Nikolaos Pappas[†], Jeongho Jeon[‡], Anthony Ephremides[‡], Apostolos Traganitis[†]
[†]Computer Science Department, University of Crete, Greece
Institute of Computer Science, Foundation for Research and Technology - Hellas (FORTH)
[‡] Department of Electrical and Computer Engineering and Institute for Systems Research
University of Maryland, College Park, MD 20742
Email: npapas@ics.forth.gr, {jeongho, etony}@umd.edu, tragani@ics.forth.gr

***Abstract*—In this paper, we evaluate the benefits of using one user of a two-user random access system to relay traffic of the other user.**

I. Introduction

Cooperative communication helps overcome fading and attenuation in wireless networks. Its main purpose is to increase the communication rates across the network and to increase reliability of time-varying links. It is known that wireless communication from a source to a destination can benefit from the cooperation of nodes that overhear the transmission. The classical single relay channel [1] exemplifies this situation. Further work on the relay channel in [2] and [3] has enabled substantial performance improvement.

However, there is evidence that additional gains can be achieved with "network-layer" cooperation (or packet-level cooperation), that is plain relaying without any physical layer considerations [4] and [5]. In this work, we focus on this type of cooperation. The work in [6] investigated the network-level cooperation in a network consisting of a source and a relay by considering the cases of full or no cooperation at the relay. A key difference between physical-layer and network-layer cooperation ideas is that the objective rate function that is maximized is the so-called stable throughput region which captures the bursty nature of traffic from the source. In [6], it was shown that the stability region of full cooperation under random-access does not always strictly contain the non-cooperative stability region.

The main contribution in this paper is to introduce the notion of partial network-level cooperation by adding a flow controller for the traffic coming to the relay from the source. We prove that the system is always better than or at least equal to the system without the flow controller. Specifically, we provide an exact characterization of the stability region of a network consisting of a source, a relay and a destination node as shown in Fig. 1. We consider the collision channel with erasures and random access of the medium. The source and the relay node have external arrivals; furthermore, the relay is forwarding part of the source node's traffic to the destination. Unlike the work in [6], the relay node is equipped with a flow controller that regulates the internal arrivals from the source based on the conditions in the network to ensure the stability of the queues. We characterize the stable throughput region under conditions of no cooperation at all, full cooperation, and probabilistic (opportunistic) cooperation. By probabilistic cooperation we mean that under certain conditions in the network, the relay may accept a packet from the source. The characterization of the stability regions is known to be challenging because the queues of the users are coupled (i.e., the service process of a queue depends on the status of the other queues). A tool that bypasses this difficulty is the stochastic dominance technique [7].

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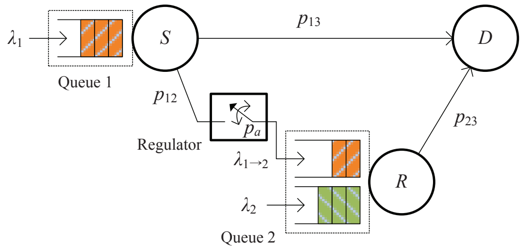

Fig. 1. Network model with regulator at the relay

II. System Model

We consider a time-slotted system in which the nodes are randomly accessing a common receiver as shown in Fig. 1. We denote with S, R, and D the source, the relay and the destination, respectively. Packet traffic originates from S and R. Because of the wireless broadcast nature, R may receive some of the packets transmitted from S and then relay those packets to D. The packets from S which failed to be received by D but were successfully received by R are relayed by R. As we impose half-duplex constraint, R can overhear S only when it is idle. Each node has an infinite size buffer for storing incoming packets, and the transmission of each packet occupies one time slot. Node R has separate queues for the exogenous arrivals and the endogenous arrivals that are relayed through R. But, we can let R to maintain a single queue and merge all the arrivals into a single queue as the achievable stable throughput region is not affected [6]. This is because

$$\mathcal{R}_1 = \left\{(\lambda_1,\lambda_2) : \frac{q_1 p_{12}(1-p_{13}) + (1-q_1)p_{23}}{q_1[p_{13}+(1-p_{13})p_{12}]}\lambda_1 + \lambda_2 < (1-q_1)p_{23}, \frac{p_{12}(1-p_{13})}{p_{12}(1-p_{13})+p_{13}}\lambda_1 + \lambda_2 < q_2(1-q_1)p_{23}\right\} \quad (1)$$

$$\mathcal{R}_1 = \left\{(\lambda_1,\lambda_2) : \frac{\lambda_1}{q_1 p_{13}} + \frac{\lambda_2}{(1-q_1)p_{23}} < 1, \quad \lambda_2 < q_2(1-q_1)p_{23}\right\} \quad (2)$$

$$\mathcal{R}_2 = \left\{(\lambda_1,\lambda_2) : \frac{(1-q_2)p_{12}(1-p_{13}) + q_2 p_{23}}{(1-q_2)\left[p_{13}+(1-p_{13})p_{12}\right]}\lambda_1 + \lambda_2 < q_2 p_{23}, \quad \lambda_1 < q_1(1-q_2)\left[p_{13}+(1-p_{13})p_{12}\right]\right\} \quad (3)$$

$$\mathcal{R}_2^{'} = \left\{(\lambda_1,\lambda_2) : \frac{\lambda_1}{(1-q_2)p_{13}} + \frac{\lambda_2}{q_2 p_{23}} < 1, \quad \lambda_1 < q_1(1-q_2)p_{13}\right\} \quad (4)$$

$$\mathcal{R}_2^{''} = \{(\lambda_1,\lambda_2) : \lambda_1 + \lambda_2 < q_1(1-q_2)p_{13} + q_2 p_{23}(1-q_1), \quad q_1(1-q_2)p_{13} \le \lambda_1 < q_1(1-q_2)[p_{13}+(1-p_{13})p_{12}]\} \quad (5)$$

the link quality between R and D is independent of which packet is selected for transmission.

The packet arrival processes at S and R are assumed to be Bernoulli with rates λ_1 and λ_2, respectively, and are independent of each other. Node R is equipped with a flow controller that regulates the rate of endogenous arrivals from S by randomly accepting the incoming packets with probability p_a; that is, it controls the *amount of cooperation* that it is willing to provide. In each time slot, nodes S and R attempt to transmit with probabilities q_1 and q_2, respectively, if their queues are not empty. Decisions on transmission are made independently among the nodes. We assumed collision channel with erasures in which, if both S and R transmit in the same time slot, a collision occurs and both transmissions fail. The probability that a packet transmitted by node i is successfully decoded at node $j(\neq i)$ is denoted by p_{ij} which is the probability that the signal-to-noise-ratio (SNR) over the specified link exceeds a certain threshold for the successful decoding. These erasure probabilities capture the effect of random fading at the physical layer. The probabilities p_{13}, p_{23}, and p_{12} denote the success probabilities over the link $S-R$, $R-D$, and $S-R$, respectively. Node R has a better channel to D than S, that is $p_{23} > p_{13}$.

The cooperation is performed at the protocol level as follows. When S transmits a packet, if D decodes the packet successfully, it sends an ACK and the packet exits the network; if D fails to decode the packet but R does and the flow controller decides to relay the packet, then R sends an ACK and takes over the responsibility of delivering the packet to D by placing it in its queue. If neither D nor R decode (or if R does not store the packet), the packet remains in S's queue for retransmission. The ACKs are assumed to be error-free, instantaneous and broadcasted to all relevant nodes.

Denote by Q_i^t the length of queue i at the beginning of time slot t. Based on the definition in [8], the queue is said to be *stable* if

$$\lim_{t\to\infty} Pr[Q_i^t < x] = F(x) \text{ and } \lim_{x\to\infty} F(x) = 1$$

Loynes' theorem [9] states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate, then the queue is stable. If the average arrival rate is greater than the average service rate, then the queue is unstable and the value of Q_i^t approaches infinity almost surely. The stability region of the system is defined as the set of arrival rate vectors $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$ for which the queues in the system are stable.

III. Main Results

This section describes the stability region for the system presented in the previous section and depicted in Fig. 1. The relay node R is equipped with a flow controller, and the parameter p_a of the flow controller is the probability to accept for the received packet from the source S. So, our objective is to find the optimum value of p_a denoted by p_a^* which maximizes the stability region. This value reflects the cooperation degree that maximizes the stability region.

Theorem 3.1: The stability region of the opportunistic cooperative network depicted in Fig. 1 is described by:

$$\mathcal{R} = \mathcal{R}_1 \bigcup \mathcal{R}_2 \quad (6)$$

- The subregion $\mathcal{R}_1$ is described as follows:
  - if $q_1 < \frac{p_{23}}{p_{13}+p_{23}}$, then $p_a^* = 1$ and the region is given by Eq.(1).
  - if $q_1 \ge \frac{p_{23}}{p_{13}+p_{23}}$, then $p_a^* = 0$ and the region is given by Eq.(2).
- The subregion $\mathcal{R}_2$ is described as follows:
  - if $q_2 \ge \frac{p_{13}}{p_{13}+p_{23}}$, then $p_a^* = 1$ and the region is given by Eq.(3).
  - if $q_2 < \frac{p_{13}}{p_{13}+p_{23}}$, then the subregion $\mathcal{R}_2$ is $\mathcal{R}_2 = \mathcal{R}_2^{'} \bigcup \mathcal{R}_2^{''}$ where:
    * if $\lambda_1 < q_1(1-q_2)p_{13}$, then $p_a^* = 0$ and the region is given by Eq.(4).
    * if $\lambda_1 \ge q_1(1-q_2)p_{13}$, then $p_a^* = \frac{\lambda_1 - q_1(1-q_2)p_{13}}{q_1(1-q_2)(1-p_{13})p_{12}}$ and the region is given by Eq.(5).

Proof: The proof is given in Section IV. ■

As seen in the theorem, there are three possible optimal values of p_a. When p_a^* equals to 0 or 1, the relay rejects or accepts all the incoming traffic from the source, respectively. The more interesting case is when $q_2 < \frac{p_{13}}{p_{13}+p_{23}}$ (the relay transmission probability is less than a threshold which is a function of the channel success probabilities) and at the same time the average arrival rate at the source is $\lambda_1 \ge q_1(1-q_2)p_{13}$; in this case the optimum cooperation strategy is probabilistic routing by the relay. The incoming traffic from the source is relayed in part, meaning that the relay accepts a packet from the source with probability p_a^*, where $p_a^* = \frac{\lambda_1 - q_1(1-q_2)p_{13}}{q_1(1-q_2)(1-p_{13})p_{12}}$ $(0 < p_a^* < 1)$. The intuition behind this result is that when the relay is not attempting to transmit "very often" and at the same time, the arrival rate at the source is greater than a certain value, then the relay is cooperating only partially. *Thus, p_a^* controls the amount of cooperation.*

IV. Stability Analysis using Stochastic Dominance

The expressions for the average service rates seen by source S and relay R are given by:

$$\mu_1 = \{(1-q_2)\Pr(Q_2 \neq 0) + \Pr(Q_2 = 0)\} \times q_1\left(p_{13} + (1-p_{13})p_{12}p_a\right) \quad (7)$$

and

$$\mu_2 = q_2\left[1 - q_1 \Pr(Q_1 \neq 0)\right] p_{23} \quad (8)$$

Since the average service rate of each queue μ_1 and μ_2 depends on the queue size of the other queue, they cannot be computed directly. We bypass this difficulty by utilizing the idea of stochastic dominance [7]; that is, we first construct hypothetical dominant systems, in which one of the nodes transmits dummy packets even when its packet queue is empty. Since the queue sizes in the dominant system are, at all times, at least as large as those of the original system, the stability region of the dominant system inner-bounds that of the original system. It turns out, however, that the stability region obtained using this stochastic dominance technique coincides with that of the original system which will be discussed in detail later in this section. Thus, the stability regions for both the original and the dominant systems are the same.

A. The first dominant system: source node transmits dummy packets

In this sub-section we obtain the region $\mathcal{R}_1$ of Theorem 3.1. We consider the first dominant system, in which node S transmits dummy packets with probability q_1 whenever its queue is empty, while node R behaves in the same way as in the original system. All other assumptions remain unaltered in the dominant system. Thus, the service rate at the relay node is given by:

$$\mu_{Q_2} = q_2\left(1-q_1\right)p_{23} \quad (9)$$

To derive the stability condition for the queue in the relay node, we need to calculate the total arrival rate. There are two independent arrival processes at the relay: the exogenous traffic with arrival rate λ_2 and the endogenous traffic from S. In the dominant system, when R receives a dummy packet from S, it simply discards that packet. When the dominant system is stable, the queue at S is stable, so the departure rate of the source packets (excluding the dummy ones) is equal to the arrival rate λ_1. Denote by S_A the event that S transmits a packet and the packet leaves the queue, then:

$$\Pr(S_A) = \left[(1-q_2)\Pr(Q_2 \neq 0) + \Pr(Q_2 = 0)\right] \times \left[p_{13} + (1-p_{13})p_{12}p_a\right] \quad (10)$$

Among the packets that depart from the queue of S, some will exit the network because they are decoded by the destination directly, and some will be relayed by R. Denote by S_B the event that the transmitted packet from S will be relayed from R, then:

$$\Pr(S_B) = \left[(1-q_2)\Pr(Q_2 \neq 0) + \Pr(Q_2 = 0)\right](1-p_{13})p_{12}p_a \quad (11)$$

The conditional probability that a transmitted packet from S (dummy packets excluded) arrives at R given that the transmitted packet exits node S's queue is given by:

$$\Pr(S_B|S_A) = \frac{(1-p_{13})p_{12}p_a}{p_{13} + (1-p_{13})p_{12}p_a} \quad (12)$$

The total arrival rate at the relay node is:

$$\lambda_{Q_2} = \lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13} + (1-p_{13})p_{12}p_a}\lambda_1 \quad (13)$$

By Loyne's Theorem, the stability condition for queue 2 at node R is given by $\lambda_{Q_2} < \mu_{Q_2}$ and, thus:

$$\lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13} + (1-p_{13})p_{12}p_a}\lambda_1 < q_2\left(1-q_1\right)p_{23} \quad (14)$$

The probability that the queue is not empty can be computed by Little's theorem and is given by:

$$\Pr(Q_2 \neq 0) = \frac{\lambda_{Q_2}}{\mu_{Q_2}} = \frac{\lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13}+(1-p_{13})p_{12}p_a}\lambda_1}{q_2\left(1-q_1\right)p_{23}} \quad (15)$$

Thus, after substituting Eq.(15) into Eq.(7), the average service rate seen by S is

$$\mu_1 = \frac{q_1}{(1-q_1)p_{23}}\{\left[p_{12}p_a(1-p_{13}) + p_{13}\right](1-q_1)p_{23} - p_{12}(1-p_{13})p_a\lambda_1 - \left[p_{12}(1-p_{13})p_a + p_{13}\right]\lambda_2\} \quad (16)$$

The stability condition for queue 1 at the source node is $\lambda_1 < \mu_1$, and after some algebra, we obtain:

$$\left[1 + \frac{p_{12}p_a(1-p_{13})q_1}{(1-q_1)p_{23}}\right]\lambda_1 + \frac{q_1\left[p_{12}p_a(1-p_{13}) + p_{13}\right]}{(1-q_1)p_{23}}\lambda_2 < q_1\left[(1-p_{13})p_{12}p_a + p_{13}\right] \quad (17)$$

An important observation made in [7] is that the stability conditions obtained by using the stochastic dominance technique are not merely sufficient conditions for the stability of the original system but are sufficient and necessary conditions. The *indistinguishability* argument applies to our problem as well. Based on the construction of the dominant system, it is easy to see that the queues of the dominant system are always larger in size than those of the original system, provided they are both initialized to the same value. Therefore, given $\lambda_2 < \mu_2$, if for some λ_1, the queue at S is stable in the dominant system, then the corresponding queue in the original system must be stable; conversely, if for some λ_1 in the dominant system, the queue at node S saturates, then it will not transmit dummy packets, and as long as S has a packet to transmit, the behavior of the dominant system is identical to that of the original system because the dummy packet transmissions are increasingly rare as we approach the stability boundary. Therefore, we can conclude that the original system and the dominant system are indistinguishable at the boundary points.

Now we will find the value of p_a that maximizes λ_1. After replacing λ_1 with y and λ_2 with x we have:

$$y = \frac{-q_1\left[p_{13}+(1-p_{13})p_{12}p_a\right]}{q_1p_{12}(1-p_{13})p_a+(1-q_1)p_{23}}x + \frac{q_1\left[p_{13}+(1-p_{13})p_{12}p_a\right](1-q_1)p_{23}}{q_1p_{12}(1-p_{13})p_a+(1-q_1)p_{23}} \quad (18)$$

when

$$0 \leq x \leq q_2(1-q_1)p_{23} - \frac{p_{12}p_a(1-p_{13})}{p_{13}+(1-p_{13})p_{12}p_a}y \quad (19)$$

After differentiating y with respect to p_a, we have

$$\frac{dy}{dp_a} = \left(\frac{A}{B}\right)' = \frac{A'B-AB'}{B^2} \quad (20)$$

where $B = q_1p_{12}(1-p_{13})p_a + (1-q_1)p_{23}$ and

$$A'B - AB' = (1-p_{13})p_{12}q_1(x-p_{23}+p_{23}q_1) \times (p_{13}q_1 - p_{23} + q_1p_{23}) \quad (21)$$

From Eq.(9), it is obvious that $x-p_{23}+p_{23}q_1 < 0$. If $p_{13}q_1 - p_{23} + p_{23}q_1 < 0$, then we have that $q_1 < \frac{p_{23}}{p_{13}+p_{23}}$. Then, $\frac{dy}{dp_a} > 0$ and y is an increasing function of p_a and, thus $p_a^* = 1$. Then, Eq.(19) becomes

$$0 \leq x \leq q_2(1-q_1)p_{23} - \frac{p_{12}(1-p_{13})}{p_{13}+(1-p_{13})p_{12}}y \quad (22)$$

and Eq.(18) becomes

$$y = \frac{-q_1\left[p_{13}+(1-p_{13})p_{12}\right]}{q_1p_{12}(1-p_{13})+(1-q_1)p_{23}}x + \frac{q_1\left[p_{13}+(1-p_{13})p_{12}\right](1-q_1)p_{23}}{q_1p_{12}(1-p_{13})+(1-q_1)p_{23}} \quad (23)$$

The stability region for this case is given by Eq.(1). If $q_1 > \frac{p_{23}}{p_{13}+p_{23}}$, it follows that $\frac{dy}{dp_a} < 0$ and, thus, y is a decreasing function of p_a and $p_a^* = 0$. Then Eq.(19) becomes

$$0 \leq x \leq q_2(1-q_1)p_{23} \quad (24)$$

and Eq.(18) becomes

$$y + \frac{q_1p_{13}}{(1-q_1)p_{23}}x = q_1p_{13} \quad (25)$$

The stability region is given by Eq.(2).

B. The second dominant system: relay node transmits dummy packets

In this sub-section we obtain the region $\mathcal{R}_2$ of Theorem 3.1. We consider the second dominant system, in which node R transmits dummy packets with probability q_2 whenever its queue is empty, while node S behaves in the same way as in the original system. All the other assumptions remain unaltered in the dominant system. The service rate for the source node is

$$\mu_1 = q_1(1-q_2)\left[p_{13}+(1-p_{13})p_{12}p_a\right] \quad (26)$$

Thus, queue 1 is stable if

$$\lambda_1 < q_1(1-q_2)\left[p_{13}+(1-p_{13})p_{12}p_a\right] \quad (27)$$

The probability that the queue is not empty is:

$$\Pr(Q_1 \neq 0) = \frac{\lambda_1}{\mu_1} = \frac{\lambda_1}{q_1(1-q_2)\left[p_{13}+(1-p_{13})p_{12}p_a\right]} \quad (28)$$

The total arrival rate at the relay node is given by:

$$\lambda_{Q_2} = \lambda_2 + \Pr(S_B|S_A)\lambda_1 \quad (29)$$

where S_A and S_B are defined in the previous sub-section. Note that $\Pr(S_A) = (1-q_2)(p_{13}+(1-p_{13})p_{12}p_a)$, $\Pr(S_B) = (1-q_2)(1-p_{13})p_{12}p_a$ and, thus, we have $\Pr(S_B|S_A) = \frac{(1-p_{13})p_{12}p_a}{p_{13}+(1-p_{13})p_{12}p_a}$. From the above it follows that the total arrival rate at the relay node is:

$$\lambda_{Q_2} = \lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13}+(1-p_{13})p_{12}p_a}\lambda_1 \quad (30)$$

The service rate for the relay node is:

$$\mu_{Q_2} = q_2\left[1-q_1\Pr(Q_1 \neq 0)\right]p_{23} \quad (31)$$

Thus, from Loyne's stability criterion, it follows that the queue is stable if $\lambda_{Q_2} < \mu_{Q_2}$ and, thus:

$$\lambda_2 + \frac{(1-p_{13})p_{12}p_a}{p_{13}+(1-p_{13})p_{12}p_a}\lambda_1 < q_2\left[1-q_1\Pr(Q_1 \neq 0)\right]p_{23} \quad (32)$$

After some algebra, we obtain:

$$\lambda_2 + \frac{(1-q_2)(1-p_{13})p_{12}p_a + q_2p_{23}}{(1-q_2)\left[p_{13}+(1-p_{13})p_{12}p_a\right]}\lambda_1 < q_2p_{23} \quad (33)$$

The indistinguishability argument at saturations holds here as well. Next we find the value of p_a that maximizes λ_2. After replacing λ_1 with x and λ_2 with y we have:

$$y + \frac{(1-q_2)p_{12}(1-p_{13})p_a + q_2p_{23}}{(1-q_2)\left[p_{13}+(1-p_{13})p_{12}p_a\right]}x = q_2p_{23} \quad (34)$$

After differentiating y with respect to p_a we have

$$\frac{dy}{dp_a} = \left(\frac{A}{B}\right)' = \frac{A'B-AB'}{B^2} \quad (35)$$

where:

$$A'B - AB' = xp_{12}(1-p_{13})(1-q_2)(p_{13}q_2 - p_{13} + q_2p_{23}) \quad (36)$$

If $p_{13}q_2 - p_{13} + q_2p_{23} > 0$, it follows that $\frac{p_{13}}{p_{13}+p_{23}} < q_2 < 1$ and y increases; thus $p_a^* = 1$ and, therefore:

$$x < q_1(1-q_2)\left[p_{13}+(1-p_{13})p_{12}\right] \quad (37)$$

and

$$y + \frac{(1-q_2)p_{12}(1-p_{13}) + q_2p_{23}}{(1-q_2)\left[p_{13}+(1-p_{13})p_{12}\right]}x = q_2p_{23} \quad (38)$$

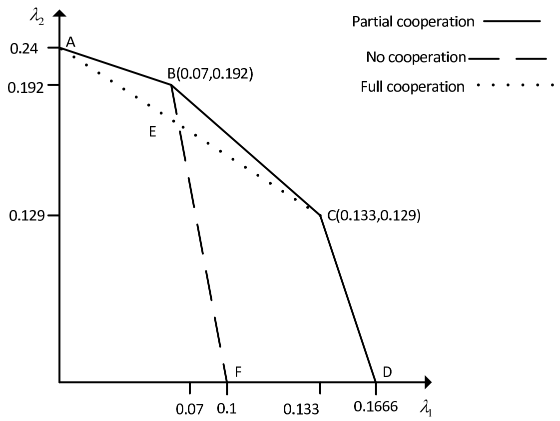

Fig. 2. Illustration of the stability region ($q_1 = 0.2$, $q_2 = 0.3$, $p_{13} = 0.5$, $p_{12} = 0.9$ and $p_{23} = 0.8$)

The stability region is then given by Eq.(3). If $q_2 < \frac{p_{13}}{p_{13}+p_{23}}$, it follows that y decreases and thus $p_a^* = 0$, hence:

$$x < q_1(1-q_2)p_{13} \tag{39}$$

and

$$y + \frac{q_2 p_{23}}{(1-q_2)p_{13}} x = q_2 p_{23} \tag{40}$$

The stability region is then given by Eq.(4).

If $x \geq q_1(1-q_2)p_{13}$ and $x \leq q_1(1-q_2)\left[p_{13} + (1-p_{13})p_{12}p_a\right]$, it follows from Eq.(27) that $p_a \geq \frac{x - q_1(1-q_2)p_{13}}{q_1(1-q_2)(1-p_{13})p_{12}}$ and, thus we obtain that:

$$p_a^* = \frac{x - q_1(1-q_2)p_{13}}{q_1(1-q_2)(1-p_{13})p_{12}} \tag{41}$$

and

$$x + y = p_{23}q_2 + q_1(1-q_2)p_{13} - q_1 q_2 p_{23} \tag{42}$$

Finally, since $0 \leq p_a \leq 1$ we have that

$$x \leq q_1(1-q_2)\left[p_{13} + (1-p_{13})p_{12}\right] \tag{43}$$

and the stability region is given by Eq.(5).

V. Numerical Results

In this section, we obtain the stability region for the three cases of no-cooperation, full cooperation and partial cooperation and we compare them in a numerical illustration where $p_a < 1$. We let $q_1 = 0.2$, $q_2 = 0.3$, $p_{13} = 0.5$, $p_{12} = 0.9$, and $p_{23} = 0.8$. In Fig. 2, we show the stability regions for the three cases. The region of partial cooperation contains the regions of the other cases. The boundaries of the stability region for the partial cooperation scheme is described by the line segments $ABCD$, and contains the region of non-cooperation (ABF) and the full cooperation (ACD). The triangular area BEC in Fig. 2 is achieved only by the partial cooperation scheme, showing that this scheme is superior compared to the rest schemes.

The line segment AB belongs to the stability region of both no-cooperation and partial cooperation schemes. It is the boundary when $\lambda_1 \leq 0.07$ (which is the average arrival rate at the source) and, corresponds to the scheme of no-cooperation or when $p_a^* = 0$. The line segment CD is the boundary for the stability region for full cooperation and partial cooperation with $p_a^* = 1$ schemes when $0.133 \leq \lambda_1 < 0.1666$. The most interesting case is the BC segment. This boundary is achieved only by the partial cooperation scheme. The value of p_a^* that achieves the boundary is $p_a^* = \frac{\lambda_1 - 0.07}{0.063}$, as $0.07 < \lambda_1 < 0.133$. In this case the relay, through the flow controller, regulates the endogenous traffic from the source by randomly accepting (with p_a^*) the packets from the source. Note that as λ_1 increases (in the interval $(0.07, 0.133)$) so does p_a^*. The values of p_a^* that achieve the boundaries of the regions are given in Table I.

TABLE I
THE VALUES OF p_a^*

Line	p_a^*
AB	0
BF	0
AC	1
CD	1
BC	$\frac{\lambda_1 - 0.07}{0.063}$

The intuition behind these results, is that when the traffic level at the source is relatively low, the optimal scheme for the relay is not to cooperate at all. When the traffic level at the source is high, the best scheme is to fully cooperate, Finally, when the source has an intermediate level of traffic, the optimal scheme is to partially offer relay services.

VI. Conclusion

We introduced the notion of partial network-level cooperation by assuming a flow controller for the endogenous traffic to the relay from the source node of the network in Fig. 1. We provided an exact characterization of the stability region for this network. We proved that the system with the flow controller is always better than or at least equal to the system without the flow controller. The flow controller regulates the degree of cooperation offered by the relay.

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